Buyers, Sellers and Middlemen: Variations on Search-Theoretic Themes*

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Abstract

We study bilateral exchange, both direct trade and indirect trade that happens through chains of intermediaries or middlemen. We develop a model of this activity and present applications. This illustrates how, and how many, intermediaries get involved, and how the terms of trade are determined. We show how bargaining with one intermediary depends on upcoming negotiations with downstream intermediaries, leading to holdup problems. We discuss the roles of buyers and sellers in bilateral exchanges, and how to interpret prices. We develop a particular bargaining solution and relate it to other solutions. In addition to contrasting our framework with other models of middlemen, we discuss the connection to different branches of search theory. We also illustrate how bubbles can emerge in intermediation.

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"You sell your own works directly, Mr Nelson?" Siobhan asked.
"Dealers have got the market sewn up," Nelson spat. "Bloodsucking bastards that they are ..." *Resurrection Men* (1991) by Ian Rankin

1 Introduction

We study bilateral exchange, both direct trade, and indirect trade that happens through intermediaries, or middlemen. We develop a model of this activity and present several applications. The framework illustrates how, and how many, middlemen get involved. Although there is much economic research on the topic, in general, a neglected aspect that seems important to business practitioners is that there are often *multiple middlemen* engaged in getting goods from the originator to end user – e.g., from farmer to broker to distributor to retailer to consumer.[1] A feature we emphasize is that the terms of trade one might negotiate with an intermediary depend on upcoming negotiations with the second, third and other downstream intermediaries. We call this *bargaining with bargainers*. We also have something to say about the roles of buyers and sellers – in particular, which are which – in bilateral exchange, and about the interpretation of prices. We develop a particular bargaining solution and discuss how it relates to other solutions. Additionally, we illustrate how *bubbles* can emerge in the value of inventories as they get traded across intermediaries.

In terms of related work, it was not so long ago that Rubinstein and Wolinsky (1987) motivated their paper as follows:

> Despite the important role played by intermediation in most markets, it is largely ignored by the standard theoretical literature. This is because a study of intermediation requires a basic model that describes explicitly the trade frictions that give rise to the function of intermediation. But this is missing from the standard market models, where the actual process of trading is left unmodeled.

[1] As a special case of this example, taken from Cooke (2000), consider illegal drugs. As another example, Ellis (2009) describes the internet like this: "If a majority of the wholesale companies being advertised are not true wholesale companies, then what are they and where are they getting their products? They are likely just middleman operating within a chain of middleman. A middleman chain occurs when a business purchases its resale products from one wholesale company, who in turn purchases the products from another wholesale company, which may also purchase the products from yet another wholesale company, and so on."

Although we think there is more to be done, several subsequent studies have attempted to rectify the situation by analyzing the roles of middlemen, and how they affect the quality of matches, the time required to conduct transactions, the variety of goods on the market, bid-ask spreads, and other phenomena.

Rubinstein and Wolinsky (1987) themselves focus on search frictions, and for them, middlemen are agents who have an advantage over the original suppliers in the rate at which they meet buyers (a different but related recent analysis can be found in Ennis 2009). Focusing instead on information frictions, Biglaiser (1993) and Li (1998,1999) present models where middlemen are agents with expertise that allows them to distinguish high- from low-quality goods, and show how the presence of informed intermediaries helps to ameliorate lemons problems. In other papers (Camera 2001, Johri and Leach 2002, Shevchenko 2004, Smith 2004, Dong 2009, and Watanabe 2010a,b), middlemen hold inventories of either more, or more types of, commodities that help buyers obtain their preferred goods more easily. See also Kalai, Postlewaite and Roberts (1978), Rust and Hall (2003), Masters (2007,2008), Tse (2009) and Bose and Sengupta (2010).

In general, middlemen may either hold inventories, or act as market makers whose role is to get traders together, without buying and selling themselves (Yavas 1992,1994,1996; Gehrig 1993). Models of these activities in financial markets include Duffie, Garleanu and Pedersen (2005), Miao (2006), Weill (2007), Lagos and Rocheteau (2009), and Lagos, Rocheteau and Weill (2009). Many of these applications can be considered part of the New Monetarist economics surveyed by Nosal and Rocheteau (2010) and Williamson and Wright (2010a,b), defined by an endeavor to explicitly model the exchange process, and institutions that facilitate this process, like money, intermediaries, etc., often using search theory. We say more later about the relationship between intermediation and money; for now, we mention that early search-based models of monetary exchange like Kiyotaki and Wright (1989) not only make predictions about which objects might emerge as media of exchange, as a function of the objects' properties and agents' beliefs, they also make perhaps-less-well-known predictions about which agents emerge as middlemen.

Search theory is the right tool for analyzing intermediaries and related institutions because, as Rubinstein and Wolinsky suggest, it models exchange explicitly. In a sense, this study is as much about search theory as it is about the substantive topic of middlemen. We set up our environment differently, in several ways, from previous studies. This is because we are less interested in *why* middlemen have a role, in the sense that much of the analysis involves circumstances where trade must be intermediated. Instead, we focus on equilibrium exchange patterns, with potentially long chains of intermediation, and the determination of the terms of trade. Still, for comparison, we present a version that generalizes standard models like Rubinstein and Wolinsky (1987). More ambitiously, we not only contrast our framework with other models of middlemen, we also discuss the connection to other branches of search theory outside the middlemen literature, in order to bridge some gaps and show how several ostensibly different models can be interpreted in light of our framework. This leads us to discuss several questions about models of bilateral trade, including, who is the buyer and who is the seller, and what is the price? In terms of price determination, we focus on a particular bargaining solution that we think is attractive, but we also compare results for other standard bargaining solutions.

A recently popular activity that our model captures is *flipping* (see Bayer, Geissler add Roberts 2011 for a recent empirical study) According to Wikipedia involves "purchasing a revenue-generating asset and quickly reselling (or 'flipping') it for profit." Although one can flip any asset, the moniker is most often applied to real estate (or sometimes IPO's). In particular, as regards our focus on intermediation chains, "Under the multiple investor flip, one investor purchases a property at below-market value, assigns or sells it quickly to a second investor, who subsequently sells it to the final consumer, closer to market value." Of course, "Profits from flipping real estate come from either buying low and selling high (often in a rapidly-rising market), or buying a house that needs repair and fixing it up before reselling." We focus on the former, although it is easy to extend the model to the case where intermediaries add value. Moreover, it is common to think that this activity may have something to do with the generation of housing (and other) price bubbles, defined in our model as equilibria where the price of inventories differs from fundamental value. As we said, the model is capable of generating bubble-like equilibria.

The rest of the paper is organized as follows. Section 2 lays out some basic assumptions and examples. Section 3 presents our extension of standard models. Section 4 goes into detail concerning the dynamics of exchange and pricing in intermediated markets. Section 5 discusses bigger issues of interpretation in this class of models, including the relation between distinct branches of the search literature. Section 6 takes up bubbles. Section 7 concludes.

2 The Basic Model

2.1 General Assumptions

Consider a set of agents $\mathcal{A} = \{A_1, A_2, ...A_N\}$, where $N \leq \infty$. They are spatially separated with the following connections: A_n can meet, and hence trade, with A_{n-1} and A_{n+1} but no one else. We can represent the population as a *graph* with the set of nodes $\mathcal{A}$ connected as show in Figure 1. There are search frictions, which means it can take time and other resources for A_n to meet A_{n+1}. There is an indivisible object x in fixed supply, and a divisible object y that anyone can produce at unit cost (i.e., the utility of producing y units of this object is $-y$). Only A_1 is endowed with x, and he can either try to trade it to A_2 in exchange for y_1, or consume it himself for utility γ_1. Hence, γ_1 is A_1's opportunity cost of trading (in Section 3, we also consider production cost).

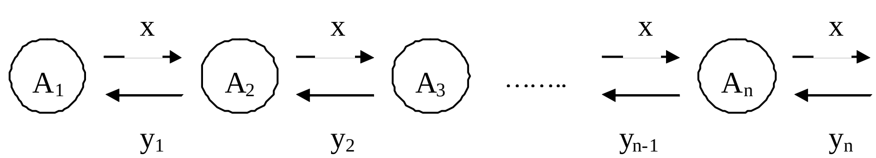


Figure 1: Population graph

More generally, if any agent A_n acquires x from A_{n-1}, he can either consume it for payoff γ_n, or try to trade it to A_{n+1} for payoff $u(y_n) = y_n$. If A_1 trades x to A_2 and A_2 trades it to A_3 ... before some A_N eventually consumes it, we say trade is intermediated and call

$A_2, ...A_{N-1}$ intermediaries or middlemen (in principle A_n could also try to trade x back to A_{n-1} but this never happens). For most of what we do it is assumed that A_n exits the market after trading x to A_{n+1}. If one wants to keep the economy going forever, one replace every A_n with a "clone" of himself after he leaves the market, as if often done in search theory. As an alternative to "cloning" in Section 3 we "recycle" agents by allowing them to continue rather than exit after trade.

2.2 Example: $N = 2$

Consider an economy with $N = 2$ – or, equivalently, for this exercise, $N = \infty$ with $\gamma_n > 0$ for $n \leq 2$ and $\gamma_n = 0$ for all $n > 2$, since this implies x will never be traded beyond A_2 (see below). In this case there can be no middlemen, but it is still useful as a vehicle to illustrate our trading protocol and as an input into the more interesting cases to follow. We begin by ignoring search, and asking what happens if A_1 happens to meet A_2. If $\gamma_2 \leq \gamma_1$, there are no gains from trade, and A_1 consumes x. If $\gamma_2 > \gamma_1$, they play the following game:

Stage 1: A_1 moves by making an offer "give me y_1 for x."

Stage 2: A_2 moves by accepting or rejecting, where:

- accept means the game ends;
- reject means we go to stage 3.

Stage 3: Nature moves (a coin toss) with the property that:

- with probability θ_1, A_1 makes A_2 a take-it-or-leave-it offer;
- with probability $1 - \theta_1$, A_2 makes A_1 a take-it-or-leave-it offer.

Figure 2 shows the game tree.[2] If the initial offer y_1 is accepted, A_1 gets payoff y_1 and A_2 gets $\gamma_2 - y_1$. If y_1 is rejected, with probability θ_1, A_1 gets the whole surplus leaving A_2

[2] We are not sure of the original use of this extensive form, but it is obviously related to Stahl (1972), Rubinstein (1982), Binmore (1987) and McCleod and Malcomson (1993), to name a few. The exact specification, with just two rounds of bargaining, where the second has a coin toss to determine who makes the final offer, appeared in early versions of Cahuc, Postel-Vinay and Robin (2006), but they ultimately switched to a more standard game that gives the same results in their application. We say more about comparing

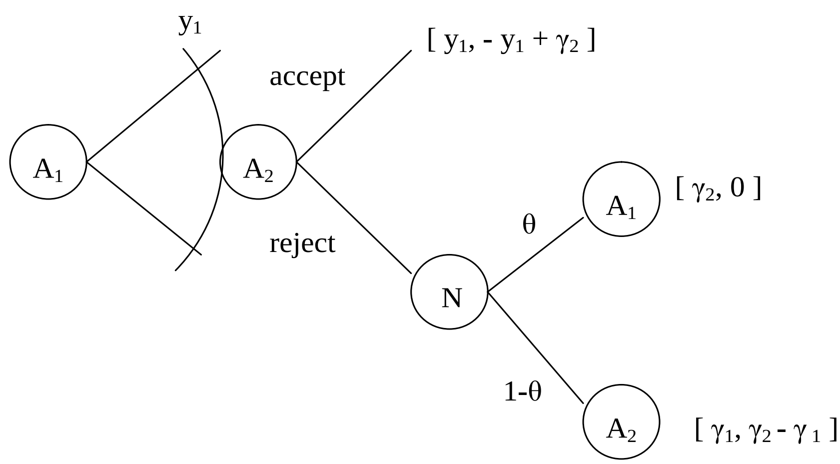


Figure 2: Game tree

with his outside option 0, and with probability $1-\theta_1$, A_1 gets his outside option γ_1 while A_2 gets the surplus $\gamma_2-\gamma_1$. The unique subgame perfect equilibrium is: at stage 1, A_1 makes A_2 his reservation offer, which means A_2 is indifferent between accepting and rejecting, and he accepts.[3] The indifference condition is $\gamma_2 - y_1 = (1-\theta_1)(\gamma_2-\gamma_1)$, or

$$y_1 = (1-\theta_1)\gamma_1 + \theta_1\gamma_2. \tag{1}$$

Payoff for A_1 is $\gamma_1+\theta_1(\gamma_2-\gamma_1)$ and that for A_2 is $(1-\theta_1)(\gamma_2-\gamma_1)$. Now, agents are not compelled to participate, but as long as $\gamma_2>\gamma_1$ we have $\gamma_1+\theta_1(\gamma_2-\gamma_1)\geq\gamma_1$ and $(1-\theta_1)(\gamma_2-\gamma_1)\geq 0$, so their payoffs beat their outside options. Equivalently, defining the total surplus as the sum of payoffs minus outside options, $S_{12}=[\gamma_1+\theta_1(\gamma_2-\gamma_1)]-\gamma_1+[(1-\theta_1)(\gamma_2-\gamma_1)]=\gamma_2-\gamma_1$, they trade as long as $S_{12}\geq 0$.

bargaining solutions when we consider nonlinear utility. With linearity, one could skip the first round and just use a coin toss to determine who makes a take-it-or-leave-it offer (e.g., as in Gale 1990 or Mortensen and Wright 2002). But since a coin toss induces risk, with risk averion, skipping the first round is not bilaterally efficient, and agents in the model prefer our game.

[3]This is almost, but not quite, right. If agents are risk neutral there is an equilibrium where the initial offer is rejected and we move to Stage 2, but it is payoff equivalent. One can refine that away by assuming either a probability $\varepsilon>0$ of an exogenous breakdown, or a disount factor $\delta<1$, between rounds. Or, as we do below, one can assume risk aversion. In these cases the equilibrium discussed in the text is unique.

For comparison, consider the standard generalization of Nash (1950) bargaining, where threat points are given by the outside options:

$$y_1 = \arg\max_y \left(y - \gamma_1\right)^{\theta_1} \left(\gamma_2 - y\right)^{1-\theta_1} \tag{2}$$

It is easy to see that the outcome is equivalent to (1). Thus this game implements the Nash solution. It also implements Kalai's (1977) proportional bargaining solution, which has become popular in search theory recently (e.g., see Lester, Postlewaite and Wright 2010 for an application and references), since it is the same as Nash in this example, giving A_1 a fraction θ_1 of S_{12}. We call the probability θ_n the bargaining power of A_n when he plays with A_{n+1}, and allow θ_n to vary across agents because we believe it is an important element of intermediation activity – e.g., one reason that athletes, artists, etc. employ agents may have to do with comparative advantages in bargaining.[4]

If it takes time and effort to meet A_{n+1}, the value of search for A_n is V_n, satisfying the flow dynamic programing equation

$$rV_n = \alpha_n \left(y_n - V_n\right) - c_n,$$

where r is the rate of time preference, α_n is a Poisson arrival rate, c_n is a flow search cost. Since c_n is only paid when A_n has x and is looking for A_{n+1}, not when A_{n-1} is looking for A_n, we can interpret it as an inventory or storage cost. In any case, we have

$$V_n = \frac{\alpha_n y_n - c_n}{r + \alpha_n}, \tag{3}$$

and A_n is willing to search for A_{n+1} only if this exceeds his opportunity cost γ_n, or

$$\left(r + \alpha_n\right)\gamma_n \leq \alpha_n y_n - c_n. \tag{4}$$

By virtue of (4) and (1), search by A_1 is viable iff

$$c_1 + \left(r + \alpha_1\right)\gamma_1 \leq \alpha_1 \left[(1 - \theta_1)\gamma_1 + \theta_1\gamma_2\right], \tag{5}$$

[4] Rubinstein and Wolinsky (1987) use a simple surplus-splitting rule, corresponding to $\theta_n = 1/2$ in our game when we have linear utility (but see below, where we have nonlinear utility). They say "The reason that we abandon the strategic approach [used in their 1985 paper] here is that it would greatly complicate the exposition without adding insights." Binmore, Rubinstein and Wolinsky (1986) provide a strong argument in favor of the strategic approach – it makes the timing, threat points, etc. less ambiguous and arbitray – and we find this clarifies rather than complicating the analysis. Whatever approach one takes, it is clearly desirable to go beyond the case $\theta_n = 1/2$ (see also Masters 2007,2008).

which says the expected payoff covers the direct search cost and opportunity cost, appropriately capitalized. Since (5) implies $S_{12} \geq 0$, the binding constraint for trade is the viability of search, not the outside options. We can let the search frictions vanish either by letting $r \to 0$ and $c_1 \to 0$, or letting $\alpha_n \to \infty$, since all that matters is r/α_n and c_n/α_n. When frictions vanish, search is viable, iff $S_{12} \geq 0$.

2.3 Example: $N = 3$

Now consider $N = 3$: an originator A_1; a potential end user A_3; and a potential middleman A_2 (or, equivalently, for this exercise, $N = \infty$ with $\gamma_n = 0$ for all $n > 3$). Note that A_3 is an end user in the sense that *if* he acquires x he consumes it, since there is no one left to take it off his hands, but it is possible that A_1 prefers consuming x rather than searching for A_2, or A_2 prefers consuming it rather than searching for A_3. Different from some related models, here A_1 and A_3 *cannot* meet directly (this is relaxed in Section 3). Hence, the only way to get x from A_1 to A_3 is via the intermediary A_2.[5]

Given these assumptions, we ask which trades occur, and at what terms. Working backwards, if A_2 with x meets A_3 then, as in the case $N = 2$, we have

$$y_2 = (1 - \theta_2)\gamma_2 + \theta_2\gamma_3. \tag{6}$$

The payoff from this trade for A_2 is y_2 and that for A_3 is $(1 - \theta_2)(\gamma_3 - \gamma_2)$, and the total surplus is $S_{23} = \gamma_3 - \gamma_2$, so they trade as long as $\gamma_3 \geq \gamma_2$. More stringently, for search by A_2 to be viable we require $V_2 \geq \gamma_2$, or

$$c_2 + (r + \alpha_2)\,\gamma_2 \leq \alpha_2\left[(1 - \theta_2)\gamma_2 + \theta_2\gamma_3\right]. \tag{7}$$

If (7) holds then, upon acquiring x, A_2 looks to trade it to A_3; if (7) fails then A_2 consumes x himself. In the latter case, A_3 is irrelevant, and effectively we have $N = 2$.

[5] Thus, we cannot ask here why the market doesn't *cut out the middlemen* – or, in more modern jargon, why there isn't *disintermediation*. On that issue, practioners say this: "why doesn't every wholesaler just buy from the manufacture and get the deepest discount? The answer is simple – not all wholesalers (or companies claiming to be wholesalers) can afford to purchase the minimum bulk-order requirements that a manufacture requires. Secondly, many manufactures only do business with companies that are established" (Ellis 2009). We do not model this explicitly, but it might be worth pursuing in future work.

So, suppose (7) holds, and back up to where A_1 meets A_2. When A_1 makes the initial offer y_1, A_2's indifference condition is $-y_1 + V_2 = (1-\theta_1)(V_2 - \gamma_1)$. Inserting V_2, we have

$$y_1 = (1-\theta_1)\gamma_1 + \theta_1 \frac{\alpha_2 y_2 - c_2}{r+\alpha_2}. \tag{8}$$

The payoff for A_1 is y_1 and that for A_2 is $(1-\theta_1)\left(\frac{\alpha_2 y_2 - c_2}{r+\alpha_2} - \gamma_1\right)$, and $S_{12} \geq 0$ iff

$$c_2 \leq -\gamma_1 (r+\alpha_2) + \alpha_2 [\theta_2\gamma_3 + (1-\theta_2)\gamma_2].$$

More stringently, for search by A_1 to be viable we require $V_1 \geq \gamma_1$, or using (6) and (8)

$$c_1 \leq -(r+\alpha_1)\gamma_1 + \alpha_1 \left\{ (1-\theta_1)\gamma_1 + \theta_1 \frac{\alpha_2 [(1-\theta_2)\gamma_2 + \theta_2\gamma_3] - c_2}{r+\alpha_2} \right\}.$$

Summarizing, after some algebra, for x to pass from A_1 to A_2 to A_3 we require

$$\begin{aligned} (r+\alpha_2)c_1 + \alpha_1\theta_1 c_2 &\leq -(r+\alpha_1\theta_1)(r+\alpha_2)\gamma_1 + \alpha_1\theta_1\alpha_2(1-\theta_2)\gamma_2 + \alpha_1\theta_1\alpha_2\theta_2\gamma_3 && (9) \\ c_2 &\leq -(r+\alpha_2\theta_2)\gamma_2 + \alpha_2\theta_2\gamma_3. && (10) \end{aligned}$$

If the inequality in (10) is reversed then A_2 consumes x if he gets it, and he gets it if

$$c_1 \leq -(r+\alpha_1\theta_1)\gamma_1 + \alpha_1\theta_1\gamma_2 \tag{11}$$

since this makes search by A_1 viable when A_2 consumes x. Note that reversing the inequality in (11) means A_1 will not search for A_2 given A_2 consumes x, but search by A_1 may still be viable if A_2, instead of consuming x, flips it to A_3. As a special case, when search frictions vanish ($r \to 0$ and $c_n \to 0$), search by A_2 is viable iff $\gamma_3 \geq \gamma_2$. Given A_2 searches, in this case, search by A_1 is viable iff $\gamma_1 \leq (1-\theta_2)\gamma_2 + \theta_2\gamma_3$. And if A_2 does not search because $\gamma_3 < \gamma_2$, then search by A_1 is viable iff $\gamma_2 \geq \gamma_1$.

To develop some more economic intuition, return to the case there are search frictions, $r > 0$ and $c_n > 0$, but now suppose $\gamma_2 = 0$ so that A_2 is a *pure* middleman, with no desire to consume x himself. If A_2 obtains x he searches for A_3 if the expected payoff exceeds the pure search cost, $\alpha_2\theta_2\gamma_3 \geq c_2$. If this inequality is reversed A_2 does not want x, and the market shuts down. But if $\alpha_2\theta_2\gamma_3 \geq c_2$, so that A_2 would search for A_3, then A_1 searches for A_2 iff

$$(r+\alpha_2)c_1 + \alpha_1\theta_1 c_2 \leq -(r+\alpha_1\theta_1)(r+\alpha_2)\gamma_1 + \alpha_1\theta_1\alpha_2\theta_2\gamma_3.$$

In words, the RHS is A_1's expected share of A_2's expected share of the end user's payoff, net of his opportunity cost, while the LHS is A_1's direct search cost and the amount he has to compensate A_2 for A_2's search costs, all appropriately capitalized.

If $\gamma_2 = 0$, $r \to 0$ and $c_n \to 0$, A_1 searches for A_2 who searches for A_3 iff $\theta_2\gamma_3 \geq \gamma_1$. The salient point is that $\gamma_3 > \gamma_1$ is *not* enough to get x from A_1 to A_3, even when γ_2, r and c_n are negligible, due to a typical holdup problem. Potential middleman A_2 knows that A_3 is willing to give anything up to γ_3 to get x, and A_1 would be willing to let it go for as little as γ_1, which sounds like there is a deal to be done. But when A_2 meets A_3 he only gets $y_2 = \theta_2\gamma_3$. He may protest he needs more just to cover his cost, $y_1 = (1-\theta_1)\gamma_1 + \theta_1\theta_2\gamma_3$. Being educated in economics, however, A_3 would (implicitly) counter that this cost is sunk and thus irrelevant in the negotiations. So A_2 will not intermediate the deal unless $\theta_2\gamma_3 \geq \gamma_1$. This is a market failure, due to lack of commitment. If A_3 and A_2 could sign a binding ex ante contract, the former could agree to pay the latter at least enough to cover his costs. Such commitment is proscribed here: as in many search models, it seems reasonable to say you cannot contract with someone before you contact someone.

Without the assumptions $r \to 0$, $c_n \to 0$ and $\gamma_2 = 0$ the results are similar but richer – e.g., there is an additional aspect of holdup as the cost c_2 is also sunk when A_2 meets A_3. Note the asymmetry: A_3 does not compensate A_2 for his search cost, but A_1 does compensate A_2 for his, because only in the former negotiations is the costs sunk; similarly, A_1 shares in the upstream value γ_3 but A_3 does not share in the downstream cost y_1. A general conclusion to draw from this is that whether exchange even gets off the ground, as well as the terms of trade and payoffs when it does, depend on not only fundamentals and bargaining power in any one bilateral trading opportunity, but also on these parameters in downstream opportunities. In particular, gains from trade between A_1 and A_2 depend on A_2's bargaining power when he later meets A_3. We call this *bargaining with bargainers*.[6]

[6] Again note that our game again implements the generalized Nash or proportional bargaining outcome, where the surplus for A_n is $y - \gamma_n$, while for A_{n+1} it is $V_{n+1} - y$ if he searches and $\gamma_{n+1} - y$ otherwise.

3 Alternative Specification

Consider a slightly different setup where, as in much of the literature, there are $N = 3$ types, but many agents of each type. Again, the types are called originators A_1, potential middlemen A_2, and potential end users A_3, with $\gamma_1 = \gamma_2 = 0$ and $\gamma_3 = \gamma > 0$. Although there is no opportunity cost of trading x for A_1 when $\gamma_1 = 0$, we allow a production cost $k \geq 0$. The key difference from our baseline model is that now anyone can meet anyone else. The main object of the analysis, in addition to determining if the market is even open, is to ascertain when middlemen are active, since A_1 has the option of trading directly with the end user A_3. Also, now agents continue in, rather than exit from, the market after a trade. Let π_n be the measure of type n agents, with $\pi_1 + \pi_2 + \pi_3 = 1$, and let $\alpha_{nn'}$ be the rate at which A_n meets $A_{n'}$.[7] Let $\sigma_n \in \{0, 1\}$ indicate whether A_n searches, $n = 1, 2$, and let $\mu \in \{0, 1\} = 1$ indicate whether A_1 trades with A_2. Let m be the steady state probability that A_2 is in possession of x, and to keep track of inventories write V_{2i} where $i \in \{0, 1\}$.

Based on all this, the steady state value of m is given by the equating the inflow and outflow of inventories, $m\alpha_{23} = (1 - m)\alpha_{21}\sigma_1\mu$. This solves for

$$m = \frac{\pi_1\alpha_{12}\sigma_1\mu}{\pi_2\alpha_{23} + \pi_1\alpha_{12}\sigma_1\mu}. \tag{12}$$

Taking m as given, the value functions satisfy

$$\begin{aligned}
rV_1 &= \alpha_{12}(1 - m)\mu[y_{12} - V_1 + \sigma_1(V_1 - k)] + \alpha_{13}[y_{13} - V_1 + \sigma_1(V_1 - k)] - c_1 &(13)\\
rV_3 &= \sigma_1\alpha_{31}(\gamma - y_{13}) + \sigma_2\alpha_{32}m(\gamma - y_{23}) &(14)\\
rV_{20} &= \sigma_1\alpha_{21}\mu(\sigma_2V_{21} - V_{20} - y_{12}) &(15)\\
rV_{21} &= \alpha_{23}(y_{23} + V_{20} - V_{21}) - c_2, &(16)
\end{aligned}$$

where $y_{nn'}$ is the bargaining outcome between n and n', and we note production cost k must be paid each time A_1 begins to search. These equations give the values of search; for $n = 1, 2$,

[7] Since the number of meetings between n and n' is the same as the number between n' and n, we have the identities $\pi_1\alpha_{12} = \pi_2\alpha_{21}$, $\pi_2\alpha_{23} = \pi_3\alpha_{32}$ and $\pi_3\alpha_{31} = \pi_1\alpha_{13}$ (one can think of the π's as primitives, putting restrictions on the α's, or vice-versa).

the value of x is then $\max\{0, V_n\}$. The indifference conditions in bargaining are

$$\gamma - y_{13} = (1-\theta_{13})[\gamma - V_1 + \sigma_1(V_1 - k)] \tag{17}$$

$$\gamma - y_{23} = (1-\theta_{23})(\gamma + V_{20} - V_{21}) \tag{18}$$

$$\sigma_2 V_{21} - V_{20} - y_{12} = (1-\theta_{12})[\sigma_2 V_{21} - V_{20} - V_1 + \sigma_1(V_1 - k)]. \tag{19}$$

A steady state equilibrium is given by a strategy profile $\mathbf{s} = (\sigma_1, \mu, \sigma_2)$ satisfying best-response conditions given below, along with m, V's and y's solving (12)-(19). The best-response conditions are: $\sigma_1 = 1$ if $V_1 > k$ and $\sigma_1 = 0$ if $V_1 < k$; $\sigma_2 = 1$ if $V_2 > 0$ and $\sigma_2 = 0$ if $V_2 < 0$; $\mu = 1$ if $S_{12} > 0$ and $\mu = 0$ if $S_{12} < 0$, where $S_{12} = \sigma_2 V_{21} - V_{20} - V_1 + \sigma_1(V_1 - k)$. Note that for A_1 to search in steady state we need $V_1 > k$, not just $V_1 > 0$. There are 8 possible pure-strategy profiles, enumerated in Table 1. For each $\mathbf{s}$ in the columns we list parameter conditions that make each element a best response, using the following notation:

$$\begin{aligned}
\bar{c}_1 &= \alpha_{13}\theta_{13}\gamma \text{ and } \tilde{c}_1 = \bar{c}_1 - k(r + \alpha_{13}\theta_{13}) \\
\bar{c}_2 &= \alpha_{23}\theta_{23}\gamma \text{ and } \tilde{c}_2 = \bar{c}_2 - k(r + \alpha_{23}\theta_{23}) \\
h(c_1) &= \tilde{c}_2 + (\tilde{c}_1 - c_1)\frac{r + \alpha_{23}\theta_{23} + \alpha_{21}(1-\theta_{12})}{\alpha_{12}\theta_{12}(1-m)} \\
g(c_1) &= \tilde{c}_2 + (\tilde{c}_1 - c_1)\frac{r + \alpha_{23}\theta_{23}}{\alpha_{12}\theta_{12}} \\
f(c_1) &= \bar{c}_2 - (\bar{c}_1 - c_1)\frac{r + \alpha_{23}\theta_{23}}{r + \alpha_{13}\theta_{13}} \\
p_1 &= \tilde{c}_1 - k\frac{r\alpha_{12}\theta_{12}(1-m)}{r + \alpha_{21}(1-\theta_{12})} \\
p_2 &= \bar{c}_2 - k\frac{\alpha_{23}\theta_{23}\alpha_{21}(1-\theta_{12})}{r + \alpha_{21}(1-\theta_{12})}.
\end{aligned}$$

	Best Response Conditions			Notes
$\mathbf{s}$	σ_1	μ	σ_2	
$(0,0,0)$	$\tilde{c}_1 \le c_1$	$\bar{c}_1 \ge c_1$	$\bar{c}_2 \le c_2$	measure 0 if $k = 0$
$(0,0,1)$	$\tilde{c}_1 \le c_1$	$f(c_1) \le c_2$	$\bar{c}_2 \ge c_2$	measure 0 if $k = 0$
$(0,1,0)$	not binding	$\bar{c}_1 \le c_1$	$\bar{c}_2 \le c_2$	
$(0,1,1)$	$g(c_1) \le c_2$	$f(c_1) \ge c_2$	$\bar{c}_2 \ge c_2$	
$(1,0,0)$	$\tilde{c}_1 \ge c_1$	not binding	$\bar{c}_2 \le c_2$	
$(1,0,1)$	$\tilde{c}_1 \ge c_1$	$\tilde{c}_2 \le c_2$	$\bar{c}_2 \ge c_2$	measure 0 if $k = 0$
$(1,1,0)$	$p_1 \ge c_1$	$k = 0$	$p_2 \le c_2$	measure 0 if $k > 0$
$(1,1,1)$	$h(c_1) \ge c_2$	$\tilde{c}_2 \ge c_2$	not binding	

Table 1: Candidate equilibria

Table 1 contains a lot of information, because it covers all 8 cases, but it is actually easy to understand. Consider $\mathbf{s} = (0, 0, 0)$, where A_1 doesn't search, A_1 wouldn't trade with A_2 if they were to meet, and A_2 wouldn't search if he had x. As Table 1 indicates, for $\sigma_1 = 0$ to be a best response we require $V_1 \le k$, which after routine algebra simplifies to $\widetilde{c}_1 \le c_1$, given other agents' strategies; for $\mu = 0$ to be a best response we require $S_{12} \le 0$, which reduces to $\bar{c}_1 \ge c_1$; and for $\sigma_2 = 0$ to be a best response we require $V_2 \le 0$, which reduces to $\bar{c}_2 \le c_2$. In the last column we mention some properties of the candidate equilibria, e.g., for $\mathbf{s} = (0, 0, 0)$ we note that $k = 0$ implies $\widetilde{c}_1 = \bar{c}_1$ and hence the best response conditions are satisfied only in the measure zero case $c_1 = \bar{c}_1$. Other cases are similar. In discussing the results here we focus on the economics, and make the algebra available on request. Basically all the economics can be seen in (c_1, c_2) space using Figure 3, drawn using the easily-verified results $h(0) > g(0)$ and $h(\bar{c}_1) = g(\bar{c}_1) = \bar{c}_2$.

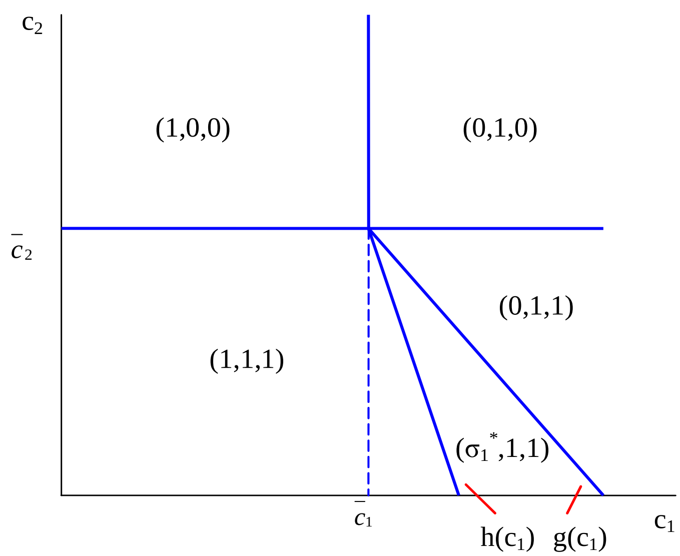


Figure 3: Equilibria with $k = 0$

To facilitate the presentation, we begin with $k = 0$, which limits the types of equilibria that can exist. First we deal with equilibria with $\sigma_1 = 0$, where A_1 does not search and the market shuts down. Even if $\sigma_1 = 0$, we must specify as part of the equilibrium whether A_1 and A_2 would trade if they were to meet, and whether A_2 would search if he acquired x, off the equilibrium path (subgame perfection). There are two possible equilibria where A_1 does not search, $\mathbf{s} = (0,1,0)$ and $\mathbf{s} =(0,1,1)$, as Figure 3 shows.[8] In each case A_1 and A_2 would trade if they were to meet, and in one A_2 would search while in the other he would not (which is a best response since in this case $y_{12} = 0$, but it never comes up in equilibrium, anyway). Naturally, these no-trade outcomes occur when c_1 and c_2 are above certain thresholds. This is hardly surprising; the point of the analysis is to determine exactly how these thresholds depend on the search and bargaining parameters, the α's and θ's, as well as the fundamental value of the good, γ.

When c_1 is lower but c_2 still high, $\mathbf{s} = (1,0,0)$ is an equilibrium, where A_1 trades with A_3 but not A_2. Middleman are inactive in this equilibrium, and A_2 would not search even if he had x. In this case, algebra implies $\sigma_1 = 1$ is a best response iff $\bar{c}_1 \geq c_1$, and $\sigma_2 = 0$ is a best response iff $\bar{c}_2 \leq c_2$. There is no intermediation because A_2 has high search costs (c_2/γ is big), meets A_3 infrequently (α_{23} is small), or bargains poorly (θ_{23} is low). When c_1 and c_2 are both low, however, as Figure 3 shows, $\mathbf{s} = (1,1,1)$ is an equilibrium, where A_1 searches and trades with A_3 or A_2, and A_2 searches for A_3 when he gets x. Now there is intermediation. Again, it is routine to check the best response conditions, which for this case are $h(c_1) \geq c_2$ and $\bar{c}_2 \geq c_2$. The diagonal line $h(c_1)$ represents how A_1's search decision depends on c_2, because y_{12} depends on A_2's expected cost and benefit from search (another instance of downstream bargaining affecting negotiations). Of particular interest is the region where $c_1 > \bar{c}_1$ and $c_2 < h(c_1)$, where the market is active only because of intermediation – i.e., if A_2 were absent all trade would cease.

This exhausts the set of possible pure-strategy equilibria.[9] As Figure 3 shows, the outcome is essentially unique – i.e., the regions do not overlap – but there is an empty area between $c_2 = h(c_1)$ and $c_2 = g(c_1)$, where pure-strategy equilibria do not exist. The problem is that when $\sigma_1 = 1$ the steady state distribution of inventories held by A_2, summarized by

[8] There are two other candidate equilibria where $\sigma_1 = 0$, $\mathbf{s} = (0,0,0)$ and $\mathbf{s} = (0,0,1)$, but they exist only for parameters in a set of measure zero.

[9] Actually, a minor technicality is that when equilibrium $(1,0,0)$ exists, so does $(1,1,0)$. It may seem strange that A_2 trades for x and then neither searches nor consumes x, but this is an equilibrium since in this case $y_{12} = 0$ and A_1 has production cost $k = 0$. Since this cannot happen for any $k > 0$, we ignore it. Any other candidate equilibrium in Table 1 exists at most for parameters in a set of measure zero.

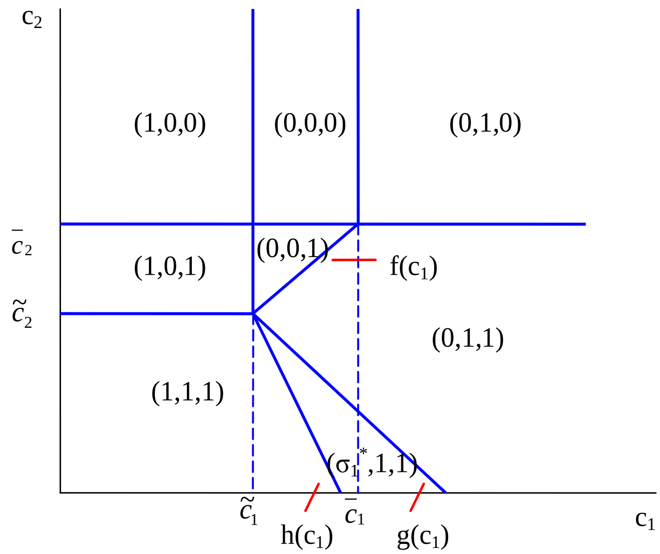


Figure 4: Equilibria with $k > 0$

m, is such that $V_1 < k$, so A_1 does not want to search, and when $\sigma_1 = 0$ the steady state m is such that $V_1 > k$, so A_1 does want to search. In Appendix A, we show how to construct the natural mixed-strategy equilibrium that fills in this region, with $\sigma_1 = \sigma_1^* \in (0, 1)$ and $V_1 = k$, meaning either A_1 agents randomizes, or some search and others do not.

This completes the analysis for $k = 0$. The results are similar, if algebraically more intense, with a production cost. Figure 4 shows the set of equilibria with $k > 0$. Compared to Figure 3, three additional types of pure-strategy equilibrium pop up in the region $[\bar{c}_1, \tilde{c}_1] \times [\bar{c}_2, \tilde{c}_2]$, which was a non-issue with $k = 0$, because then $\bar{c}_j = \tilde{c}_j$. These include two new no-trade outcomes, $(0, 0, 0)$ and $(0, 0, 1)$, plus a new active equilibrium with $(1, 0, 1)$. In the latter case, A_1 searches trades with A_3 and not A_2, but if A_2 acquired x out of equilibrium he would search for A_3. This could not happen when $k = 0$, because if $k = 0$ there are gains from trade between A_1 and A_2 iff search is viable for A_2. This completes the analysis of $k > 0$. We leave as an exercise the related extension, where there is also a production cost when A_2 acquires and retrades x, say due to improvements, as may be relevant when flipping real estate.

In terms of other substantive results, one can compare the predictions of our model in the pure-strategy equilibrium with active middlemen, $\mathbf{s} = (1, 1, 1)$, to those in Rubinstein and Wolinsky (1987).[10] As in their model, we can show that $y_{23} > y_{13} > y_{12}$, at least if we adopt their symmetric bargaining assumption. We can also show that, in general, intermediaries' profit margin, given by

$$y_{23} - y_{12} \propto \theta_{23}(r\gamma + c_2) + (1 - \theta_{12})[(\widetilde{c}_2 - c_2) + \alpha_{21}\theta_{23}(\gamma - k)] > 0,$$

is increasing in α_{23}. It is also increasing in θ_{23}, and in r at least when $k = 0$. As $r \to 0$, our margin remains positive even if $c_n = k = 0$, while in Rubinstein-Wolinsky it vanishes.

Obviously the main result in Rubinstein-Wolinsky is that middlemen are active iff they can meet end users faster than originators can, $\alpha_{23} > \alpha_{13}$. When we set $c_n = 0$, as they do, but still allow general θ's, middlemen are active iff $\alpha_{23}\theta_{23} > 0$. There are two differences. First, with symmetric bargaining of course θ cancels. More subtly, all that matters in the model is the product $\alpha_{nn'}\theta_{nn'}$.[11] Hence, the generalization of Rubinstein-Wolinsky is that middlemen are active iff $\alpha_{23}\theta_{23} > \alpha_{13}\theta_{13}$, which is still different from our condition, where the RHS is 0. This is because they assume that A_1 and A_3 exit the market, while only A_2 continues, after trade. By contrast, here all agents to continue in the market. Thus, there is no opportunity cost for our A_1 to trade with A_2, while in their setup, this would be the last trade he ever made. We can of course allow any subset of types to continue or exit after trade (details available on request), as may be appropriate for different applications. In any event, having solved this version with differences in costs, arrival rates and bargaining power, for the standard case of $N = 3$, we revert to the baseline specification to investigate the potential for longer chains of intermediation.[12]

[10]For the record, our model generates the same qualitative predictions in the mixed-strategy equilibrium with active middlemen, $\mathbf{s} = (\sigma^*, 1, 1)$, and there is no mixed-strategy equilibrium in Rubinstein-Wolinsky.

[11]This is also true in related search models, like Lagos and Rocheteau (2009), where they interpret the product as an effective arrival rate, adjusted for bargaining power.

[12]Actually, the results in this section are not that different from our baseline model, where A_n can only trade x to A_{n+1}. Indeed, a diagram similar to Figure 3 or 4 can be drawn for that model, again partitioning (c_1, c_2) space into regions, with similar if not exactly the same economic properties (e.g., one difference is that, in the baseline model, when A_2 does not search he consumes x, while here he disposes of it).

4 Multiple Middlemen

In the model where A_n can only trade x to A_{n+1}, given that the latter plans to trade it again, the equilibrium condition when A_n and A_{n+1} bargain is

$$\frac{\alpha_{n+1}y_{n+1}-c_{n+1}}{r+\alpha_{n+1}}-y_n=(1-\theta_n)\left(-\gamma_n+\frac{\alpha_{n+1}y_{n+1}-c_{n+1}}{r+\alpha_{n+1}}.\right) \tag{20}$$

Solving for y_n, we have

$$y_n=(1-\theta_n)\gamma_n+\theta_n\frac{\alpha_{n+1}y_{n+1}-c_{n+1}}{r+\alpha_{n+1}}\equiv\rho_n\left(y_{n+1}\right). \tag{21}$$

We interpret $y_n=\rho_n\left(y_{n+1}\right)$ as a best response condition for A_n: it gives y_n, his initial offer strategy when he meets A_{n+1}, as a function of others' strategies, as summarized by y_{n+1} which is all he needs to choose y_n.

Conditional on A_n having acquired x, it is easy to see that A_n and A_{n+1} end up trading in equilibrium iff:

1. $V_n=\dfrac{\alpha_n y_n-c_n}{r+\alpha_n}\geq\gamma_n$, so A_n wants to search;
2. $V_{n,n+1}=y_n\geq\gamma_n$, so A_n wants to trade;
3. $V_{n+1,n}=-y_n+V_{n+1}\geq 0$, so A_{n+1} wants to trade.

The second condition is not binding given the first, while the third reduces to[13]

$$y_{n+1}\geq\frac{\gamma_n(r+\alpha_{n+1})+c_{n+1}}{\alpha_{n+1}}. \tag{22}$$

Now, to investigate how long intermediation chains can be, consider a quasi-stationary environment, where α_n, c_n and θ_n are the same for all n, while $\gamma_n=\gamma$ for $n\leq N$, $\gamma_{N+1}=\hat{\gamma}>\gamma$, and $\gamma_n=0$ for $n>N+1$. We call A_{N+1} the end user because, if he gets x, he consumes it, since A_n does not value x for $n>N+1$.[14]

[13] By inserting $y_{n+1}=\rho_n^{-1}\left(y_n\right)$, one can see (22) holds iff $y_n\geq\gamma_n$.

[14] As above, we are asserting here that x cannot go from A_{N+1} to A_{N+2} when $A_n=0$ for all $n>N+1$. The proof is a special case of a no-bubble result given below.

Now (21) can be written $y_n = \rho(y_{n+1})$ for $n < N$, where $\rho(y) = (1-\theta)\gamma + \theta(\alpha y - c)/(r+\alpha)$, while $y_N = (1-\theta)\gamma + \theta\hat{\gamma}$. Clearly, $\rho(y)$ has a unique fixed point,

$$y^* = \frac{(1-\theta)\gamma(r+\alpha) - \theta c}{r + \alpha(1-\theta)},$$

where we assume $c < (1-\theta)\gamma(r+\alpha)/\theta$ so $y^* > 0$. See Figure 5, where (it is easy to check) $y_N > \gamma > y^* > \rho(0)$. The way to read in Figure 5 is: given A_{n+1} correctly anticipates getting y_{n+1} from A_{n+2}, in bargaining with A_n the equilibrium outcome is y_n. Now, to find equilibrium, begin by working backwards: set $y_N = (1-\theta)\gamma + \theta\hat{\gamma}$ and iterate on $y_n = \rho(y_{n+1})$ to construct a sequence $\{y_n\}$, where it is obvious that $y_n \to y^*$ as $n \to -\infty$. Then, since we are actually interested in what happens as n increases, moving forward in real time, pick a point in this sequence and iterate forward. This generates a candidate equilibrium.

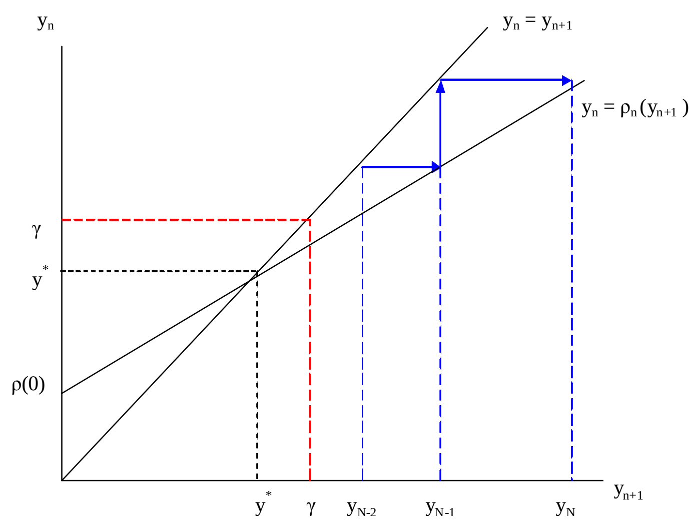


Figure 5: Path of y_n

The sequence thus constructed is still only a candidate equilibrium because we still have to check if search is viable for all agents in the chain. Clearly we cannot have arbitrarily long chains, since, going backwards in time, this would involve starting arbitrarily close to $y^* < \gamma$, and if $y_n < \gamma$ the holder of x would rather consume it than search. Consider e.g. starting with A_{N-2} holding x. Suppose A_{N-2} searches and trades x to A_{N-1}, who then searches and trades with A_N, who finally searches and trades x to the end user A_{N+1}. To see if this is

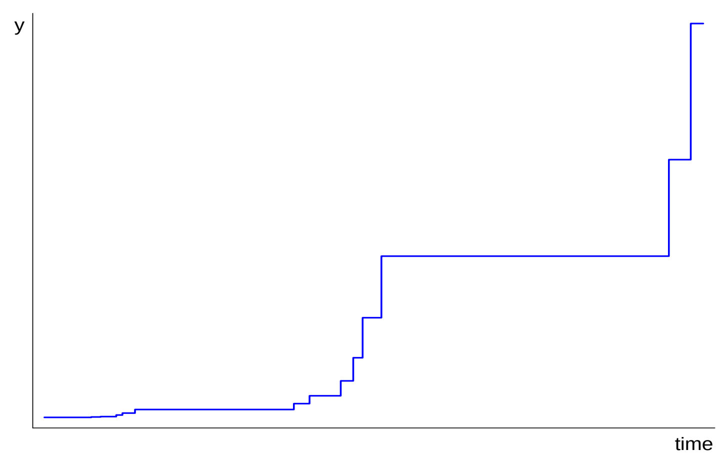


Figure 6: Typical realized path for y

viable, solve for $y_{N-2} = \rho^2(y_N)$, where $\rho^2(y) = \rho \circ \rho\,(y)$ and again $y_N = (1-\theta)\gamma + \theta\hat{\gamma}$, and check

$$V_{N-2} = \frac{\alpha\rho^2(y_N) - c}{r+\alpha} \geq \gamma.$$

If c is not too big, we can support trade with two middlemen between the originator and end user. For any $c > 0$ we cannot support trade with an arbitrary number of middlemen, however, since $y_{N-j} \to y^* < \gamma$, so there is a maximum viable chain. However, if $\gamma = c = 0$, then $y^* = 0$ and there are arbitrarily long chains starting near 0 and ending at y_N.

Even this simple model generates some interesting predictions. To illustrate, let T_n be the random date when A_n trades x to A_{n+1}. There are two striking properties of our trading process, one from economics and one from statistics. First, as is obvious from Figure 5 and reproduced in Figure 6, $\triangle y$ increases over time: as x gets closer to the end user, not only y but the increments in y rise. Second, since the underlying arrival rates are Poisson, as is well known, the interarrival times $T_n - T_{n-1}$ are distributed exponentially. This entails a high probability of short, and a low probability of long, interarrival times. Hence, typical realizations of the process have trades *clustered*, with many exchanges occurring in short intervals separated by long intervals of inactivity. This gives the distinct appearance of market frenzies interspersed by long lulls, although since Poisson arrivals are memoryless there are indubitably no frenzies or lulls in any meaningful economic sense.[15] Figure 6 illustrates these two features: the statistical property that intervals of rapid activity are interspersed by long lulls; and the economic property that y grows at an increasing rate, as it approaches the end user, to the ultimate value $y_N = (1-\theta)\gamma + \theta\hat{\gamma}$.

5 Discussion

Since the Introduction, we have refrained from using the words *buyer*, *seller* and *price*. This is intentional, as we want to raise some issues associated with such usage. First, we contend that in the analog to our model found in much of the search literature, in our notation, x represents a good and y money, and with this interpretation y is the price, the agent who trades x for y is a seller, and the one who trades y for x is a buyer. Noteworthy papers that we interpret in this way include, in addition those on middlemen discussed earlier, Diamond (1971,1987), Butters (1977), Burdett-Judd (1983) and Rubinstein-Wolinsky (1985), all of which have an indivisible object corresponding to x called a consumption good (or in some applications a production good like labor), and a divisible object y interpreted as the price (or wage). Of course, although they may think of y as dollars, these models do not literally have *money* – what they have is *transferrable utility*.[16]

[15] This is explained in any good text on stochastic processes. As Çinlar (1975, 79-80), e.g., puts it: "the interarrival times $T_1, T_2 - T_1, T_3 - T_2, ...$ are independent and identically distributed random variables, with the ... exponential distribution ... Note that this density is monotone decreasing. As a result, an interarrival time is more likely to have a length in $[0, s]$ than in a length in $[t, t+s]$ for any t. Thus, a Poisson process has more short intervals than long ones. Therefore, a plot of the time series of arrivals on a line looks, to the naive eye, as if the arrivals occur in clusters." Yet the memoryless property implies that "knowing that an interarrival time has already lasted t units does not alter the probability of its lasting another s units."

[16] Submitted in evidence, from middlemen papers, consider the following: Rubinstein-Wolinsky (1987, p.582) describe payoffs as "consumption values (in monetary terms)." Biglaiser (1993, p.213) says "Each buyer is endowed with money." Yavas (1994) describes a standard model of middleman by "The sellers and the middlemen value the good (in monetary terms) at zero, while the buyers value the good at one." Yavas (1992) is more careful, saying "In order to avoid the additional questions associated with having money in the economy, this endowment has not been labeled as money." Johri-Leach (2002) are also careful to say "units of a divisible *numeraire good* are exchanged for units of an indivisible *heterogeneous good*," although they have no problem assuming payoffs are linear in the former. Even those less cavalier about money are quick to deem who is a buyer or seller and what is the price.

In search theory outside the middleman literature, Butters (1977, p.466) says "A single homogeneous good is being traded for money," while Burdett-Judd (1983, pp.955,960) say consumers search "to lower the expected costs of acquiring, a desired commodity, balancing the monetary cost of search against its monetary benefit," while firms want to "make more money." Diamond (1971,1987) does not mention money explicitly, but he thinks of y as the price, and buyers are those with payoff $\gamma - y$ while sellers are those with payoff y.

Identifying money with (more accurately, confusing money with) transferrable utility is standard fare by even the best economic theorists. Consider Binmore (1992): "Sometimes it is assumed that contracts can be written that specify that some utils are to be transferred from one player to another ... Alert readers will be suspicious about such transfers ... Utils are not real objects and so cannot really be transferred; only physical commodities can actually be exchanged. Transferable utility therefore only makes proper sense in special cases. The leading case is that in which both players are risk-neutral and their von Neumann and Morgenstern utility scales have been chosen so that their utility from a sum of money x is simply $U(x) = x$. Transferring one util from one player to another is then just the same as transferring one dollar." Unfortunately, it ain't necessarily so – and this is about more than an abhorrence for the dubious, if evidently not discredited, practice of putting money in the utility function.

In serious monetary theory, it is *not* trivial to transfer dollars across agents, because they tend to run out. No one has an unlimited supply of cash, and for almost all inflation rates (except the Friedman rule) agents carry less than the amount required for unconstrained trade. And, in any case, payoffs are usually not linear in dollars (with some exceptions, e.g., Lagos and Wright 2005). Examples of search-based monetary theory that looks like the setup in this paper include the models in Shi (1995), Trejos-Wright (1995), Kocherlakota (1998), Wallace (2001) and many other papers. The point we emphasize is that all those models take a diametric position to the above-mentioned applications outside of monetary economics: they assume y is a consumption good and x is money. The most apparent difference is that, under one interpretation, money is divisible and consumption goods are indivisible, while under the other, money is indivisible and goods indivisible. Superficially this favors the first interpretation, since divisibility is one of the properties (along with storability, portability and recognizability) commonly associated with money. On reflection,

In several places Osborne-Rubinstein (1990) decribe models where "A single indivisible good is traded for some quantity of a divisible good ('money')." Gale (1987, p.20) more accurately says "A single, indivisible commodity is traded. Buyers and sellers have transferable utility." To sum up, with some exceptions, this literature suggests we interpret x as a good, y as a money price, A_n as a seller and A_{n+1} as a buyer.

however, we do think this should be given much weight.[17]

A better discriminating criterion stems from the functional definitions of money: it is a unit of account, a store of value, and a medium of exchange. The unit of account function – which means that American prices tend to be quoted in dollars, and European prices in euros – seems relatively uninteresting, because for anything of consequence it cannot matter much whether we measure prices in dollars or euros any more than whether we measure distance in feet or meters. Moving to the store of value function, it seems clear that in the baseline model it is actually x and not y that constitutes a store of value: x is a durable good that, when acquired by A_n at some date, enables him to enjoy a payoff y_n at a future date. The more natural interpretation of y is that it is a perishable good, or a service rather than a good, that is not carried across time but produced for immediate consumption. It is hard to imagine a perishable good or service serving as money. By contrast, x, which is kept in inventory, is obviously a storable asset.

Moreover, x satisfies the standard definition of a medium of exchange: an object that is accepted in trade not to be consumed or used in production by all who that accepts it, but instead to be traded again later. Now, in the above model x happens to be *commodity money*, since an end user ultimately does consume it for a direct payoff, as opposed to *fiat money* which does not generate a direct payoff for anyone. We have more to say about this below; for now, we simply submit that y is certainly not a medium of exchange in any of the above-mentioned search models: it is accepted by everyone for its direct payoff, and never to be traded again later. Moreover, it is exactly the classic double-coincidence problem that makes x useful here: when A_n wants y from A_{n+1}, he has nothing to offer in trade absent asset x. Indeed, x can facilitate this exchange even if A_{n+1} does not especially enjoy x, as in the case of $\gamma_{n+1} = 0$, or, for that matter, $\gamma_{n+1} < 0$, since A_{n+1} is only going to trade it again. Based on all this, in terms of calling either x or y money, maybe the monetary search

[17] One reason is that earlier contributions to the search-based monetary literature actually have both x and y indivisible, while more recent ones have them both divisible (see the surveys cited in the Introduction). Another is that, as a matter of historical record, objects used as money including coins were often less-than-perfectly divisible, with significant economic consequences (e.g., Sargent and Velde 2000).

theorists got this one right.[18]

Does it matter? While at some level one might say the issue is purely semantic, as if that were reason not to be interested, we think it actually may matter for how one uses the theory. For instance, it determines who we call the buyer or seller and what we mean by the price. To make this point, we first argue that in nonmonetary exchange – say, when A gives B apples for bananas – it is *not* meaningful to call either agent a buyer or seller. Of course, one can them whatever one likes, but then the labels buyer and seller convey nothing more than calling them A and B. However, when A gives B apples for genuine money, for dollars or euros, everyone should agree that A is the seller and B the buyer (it is perhaps less clear what to call them when A gives B euros for dollars, but that is beside the point). We identify agents who pay money as buyers and ones who receive money as sellers, and claim this corresponds to standard usage.[19]

Again, one can label objects anything one likes, and be on firm ground logically, if not aesthetically. But would anyone want to reverse the labels in, say, the Mortensen-Pissarides (1994) labor-market model, taking the agents we usually think of as workers and calling them firms, and vice versa? One could prove the same theorems, but it would make a difference when considering applied questions (e.g., should we tax/subsidize search by workers or by firms?). Using the interpretation in the previous middlemen literature, where y is money and x is a good, when A_n and A_{n+1} trade the former is the seller and the latter is the buyer. Using the interpretation in the monetary literature mentioned above, where x is money and y is a good, A_n is the buyer and A_{n+1} is the seller. If one agrees that it makes a substantive difference who we call workers and firms in the labor market, it can similarly make a difference who we call buyers and sellers in goods markets (e.g., for substantive

[18] One can also argue x plays much the same role that money plays in non-search monetary models, such as overlapping-generations models (Wallace 1980). Fleshing this out might be interesting.

[19] As evidence, consider the following definitions of the verbs *buy* and *sell*: "acquisition of an article, and legal assumption of its ownership, in exchange for money or value" and "to transfer ownership of a property in exchange for money or value" (businessdictionary.com); "to aquire possession, ownership, or rights to the use or services of by payment especially of money" and "to give up (propery) for something of value (as money)" (Meriam Webster); "to acquire the possession of, or the right to, by paying or promising to pay an equivalent, esp. in money" and "to transfer (goods) to or render (services) for another in exchange for money" (Dictionary.com); or "to get something by paying money for it" and "to give something to someone else in return for money" (Cambridge Dictionaries Online).

questions, like should we tax/subsidize shoppers or retailers?).

Moreover, the two interpretations give opposite predictions for price behavior. If we normalize the size of the indivisible x to 1, without loss of generality, under the interpretation that y is money and x is a good, the price is obviously y. But if y is the good and x is money then $1/y$ is the price, since now a normalized unit of money buys y units of the good. To see how this matters, look back at Figure 6. Using the first interpretation, theory predicts prices are rising over time, as more and more money y is required to buy the same amount of consumption x as we get closer to the end user. Using the second interpretation, prices are falling, as over time more and more consumption y can be had for the same amount of money x. For empirical work, at least, one has to make a choice. We do not propose one irreproachable interpretation, since this depends on the issues at hand. We do think it is relevant to broach the issue.

We also find it intriguing that, from a legal standpoint, it often makes a vital difference who we construe as buyer and seller. It is not uncommon to have laws or conventions that allow buyers to return goods and demand a refund, or at least store credit within a certain period of time with no questions asked – the principle of *caveat emptor* notwithstanding.[20] We are not aware of similar laws or conventions applying to sellers, and with rare exceptions like bad checks or counterfeit currency, accepting monetary payment entails finality – suggesting a more rigorous principle of *caveat venditor*. Also, in law, "There is a 'bias' in favour of buyers. Buyers are not obligated to disclose what they know about the value of a seller's property, but sellers are under a qualified obligation to disclose material facts about their own property." (Ramsay 2006). Beyond legal systems narrowly construed, is also true that private trading platforms, like ebay, have rules and regulations that treat buyers and sellers differently (e.g., Beal 2009).

[20] "The two basic types of warranties are express warranties and implied warranties. An express warranty is any representation or affirmation about the goods made by the seller's words or conduct. ... Implied warranties are warranties that are imposed on sellers by law. A warranty of merchantability is implied in every sales contract. This warranty is a promise that the goods pass without objection in the trade, are adequately packaged, conform to all promises or affirmations of fact on the container, and are fit for the ordinary purposes for which such goods are used. The implied warranty of merchantability also includes a promise that multiple goods will be of even kind and quality." From "Sales Law - Warranties" at http://law.jrank.org/pages/9989/Sales-Law-Warranties.html.

We also cannot help but comment on the asymmetric treatment of buyers and sellers in illegal markets. It is commonly understood that with drugs and prostitution, the one who receives the money is usually treated much more harshly than the one who pays (which raises a question about trading drugs directly for sex, but like trading dollars for euros, this is beside the point). Indeed, with drugs, it is so much more common to target sellers than buyers that targeting the latter is referred to as a "reverse sting" operation. With prostitution, historically buyers have almost always been treated much more leniently than sellers.[21] Similarly, where gambling and drinking are illegal, sellers are typically deemed the bigger villains. The same can be said for the markets for illegal guns, fireworks and so on. Although we are less sure about this case, it may also be true for stolen merchandise – and note that a fence is not much different from a middleman in our model. Intermediaries in general are often considered less than honorable, if not outright heinous, since they do not themselves produce anything but simply profit from others, as evidenced by our epigraph referring to "bloodsucking bastards" and by other timeless condemnations of moneylenders and the like. This is, of course, mainly out of ignorance of the idea that getting goods from A to B is a productive activity.

In any case, for what it's worth, we find it is the norm to deem sellers – those who receive money in exchange for goods – worse in legal and moral senses than buyers – those who receive goods in exchange for money. There are exceptions, as in the case of in child prostitution, but this can be explained by saying that the distinction between adults and minors takes precedence over the distinction between buyers and sellers without denying the existence of the latter. Another exception might be when a student, or, perish the thought, a professor, gives someone money to write a paper for them.[22]

[21] See the "The Staight Dope" at http://www.straightdope.com/columns/read/2774/prostitution. Of course, some argue that this is merely another manifestation of male dominance: "As long as society remains male dominated, women selling sex will be in a more vulnerable position than men buying sex." See "The swedish approach to prostitution" at http://www.sos-sexisme.org/English/swedish.htm. This suggest an obvious empirical test by looking at male prostitutes – *if* one were willing to swallow some rather dubious *ceterus paribus* assumptions.

[22] Our objective is to point out some asymmetries between buyers and sellers, not to explain them, but one can point to various rationales. On focusing enforcement on drug or gun dealers rather than buyers, it is sometimes said there are barriers to entry, at least at the high end, so that it is difficult for newcomers to replace those who are incarcerated (Koper and Reuter 1995). Yet the reverse argument can be made at the

Returning to the theory, consider again Figure 1. The usual approach to middlemen interprets x as a consumption good being passed from originator to end user by chains of intermediaries. At each link in the chain x trades for y_n units of money – or, better, for y_n transferable utils – or, better still, for some amount of a different consumption good that is produced at cost y_n by A_{n+1} and consumed for utility y_n by A_n. We have no problem with linear utility, our quibble is more about asserting that agents getting direct utility from money, or if it is interpreted as indirect utility, that the value function is linear in dollars. These issues aside, we think our framework provides a useful model of intermediation chains, where one good x trades for another good y, or even a sequence of different goods $\{y_n\}$, since it is easy to imagine A_n altering or improving x in some way before he flips it to A_{n+1}. But it is also a coherent model of agents trading an asset x in exchange for goods produced at cost y_n by A_{n+1} and consumed for utility y_n by A_n.

Also notice that if we use modulo N arithmetic, this chain becomes a circle, where A_1 consumes the output of A_2 who consumes the output ... of A_N who consumes the output of A_1. This looks much like the environment used to study money in Kiyotaki and Wright (1989), or the generalization in Aiyagari and Wallace (1991). Given that, one wonders, could the model support equilibria where x, like some commodity money, is never consumed? Or where x has no intrinsic value? Or even where it has negative intrinsic value?

6 Intermediation Bubbles

This leads us to a discussion of bubbles.[23] In the analysis so far, we could get chains of trade, and with $c_n \approx 0$ these chains can be quite long, but the equilibrium is always tied down by

low end, where a dealer removed from a streetcorner is quickly replaced. Other arguments one hears include the view that those who profit most from illegal activity should face highest risk of punishment, as should those that provide bad role models by their financial success. Another is the principle that we ethically judge actions that put others at risk more harshly than those that put oneself at risk; while this might ring true for drugs, it is difficult to make the case for prostitutes. The same might be said for the position that if B is weak and A exploits that weakness, A should be judged more harshly. There is also the idea that for many markets, legal or illegal, sellers have a lot more transactions than buyers, so it is cost effective to go after sellers. We leave this all to future research.

[23] We will define what we mean by this term presently. The bubble literature is ever expanding, to the extent that it is too big to go into here; recent papers by Farhi and Tirole (2010) and Rocheteau and Wright (2010) provide extensive references and help put our results in context.

the fact that there is a determinate end user who consumes x. Now consider the possibility that no one consumes x – it gets traded forever. As in the quasi-stationary environment, assume $\alpha_n = \alpha$, $c_n = c$ and $\theta_n = \theta$ for all n, and now also $\gamma_n = \gamma$ for all n. In this stationary environment, the best response function is

$$y_n = (1-\theta)\gamma + \theta \frac{\alpha y_{n+1} - c}{r+\alpha} = \rho(y_{n+1}), \tag{23}$$

as shown in Figure 7. The viability condition for A_n to search is $\gamma \leq (\alpha y_{n+1} - c) / (r+\alpha)$.

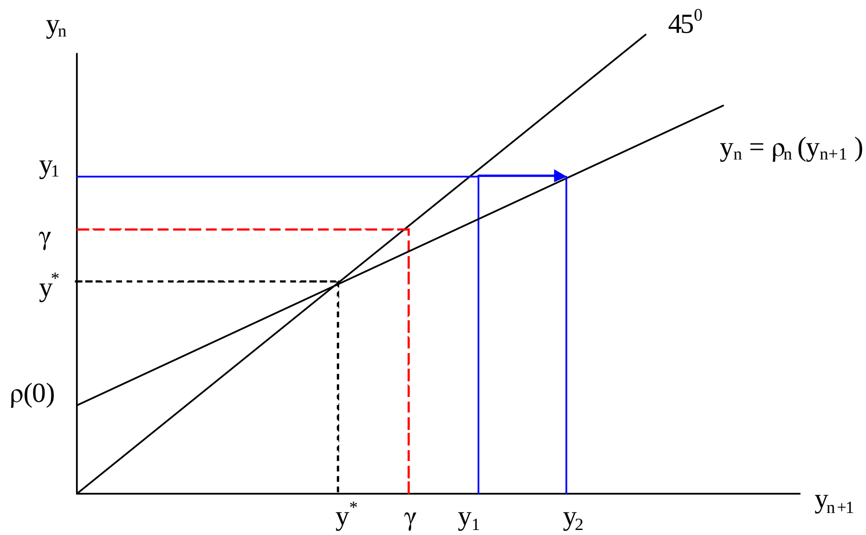


Figure 7: Best-response condition (with linear utility)

The principal difference from the quasi-stationary model, with a manifest end user A_{N+1}, is that we no longer have a terminal condition $y_N = \theta\gamma + (1-\theta)\hat{\gamma}$ to tie down the equilibrium path. Still, since $y_n = \rho(y_{n+1})$ implies $\partial y_{n+1}/\partial y_n = (r+\alpha)/\alpha\theta$, as seen in Figure 7, there is only one solution to the difference equation (23) that remains nonnegative and bounded: $y_n = y^*$ for all n. Any path that starts, e.g., at $y > y^*$ looks like a bubble: agent A_n is willing to search for A_{n+1} because he expects a high y_n, A_{n+1} is willing to give a high y_n because he expects an even higher y_{n+1}, and so on. But such explosive paths are *not* consistent with equilibrium as long as we make the standard assumption that there is some upper bound $\overline{y}$ (e.g., $\overline{y}$ could be the total output of known universe). One cannot rationally believe that y_n will grow beyond $\overline{y}$, and hence this bubble-like path for y_n is not an equilibrium. The only equilibrium is $y_n = y^*$ for all n, and since $\gamma > y^*$ it implies $V_n = (\alpha y^* - c) / (r + \alpha) < \gamma$. Hence, no one searches, since there are no gainsfrom trade when all agents attach the same value to x, at least for the model as specified so far.

If A_n trades x to A_{n+1}, off the equilibrium path, he must get exactly $y_n = \gamma$ for it, since he couldn't get more and he wouldn't take less. We call $y_n = \gamma$ the *fundamental value* of x, which is evidently what it is worth in terms of intrinsic properties. In this environment, there are no gains from trade based on fundamentals – consumption value γ, search cost c, proximately to a potential end user, and so on are constant across agents. We therefore say that a bubble exists in this situation when, in equilibium, A_n and A_{n+1} trade and this generates a strictly positive surplus. The above discussion implies that, even with $N = \infty$, in this stationary environment there are no bubbles. Indeed, trade could only occur here if there were a bubble, but this cannot be an equilibrium.[24] Even if some of the equilibrium trajectories discussed above – e.g., the one shown Figure 6 – arguably have certain earmarks that people commonly associate with bubbles, at least in the sense that y_n increases by larger and larger increments with each trade, until we hit the end user, we do not think that is remarkable. In those cases, equilibrium $\{y_n\}$ is clearly pinned down by fundamentals. We are after bigger game.

To this end, suppose we now let the utility of consuming y be $U(y)$, with $U(0) = 0$, $U' > 0$ and $U" < 0$ (keeping the cost of producing y equal to y, without loss of generality). Then $y_\gamma = U^{-1}(\gamma)$ becomes the cost to A_{n+1} of covering A_n's outside option γ, and our usual bargaining game leads to

$$y_n = (1 - \theta_n) y_\gamma + \theta_n \frac{\alpha_{n+1} U(y_{n+1}) - c_{n+1}}{r + \alpha_{n+1}} = \rho_n (y_{n+1}) .$$

In the stationary environment, in particular,

$$y_n = (1 - \theta) y_\gamma + \theta \frac{\alpha U(y_{n+1}) - c}{r + \alpha} = \rho (y_{n+1}) . \tag{24}$$

Figure 8 shows a case where $y^* > y_\gamma$, or equivalently, $U(y^*) > \gamma$, which we could *not* get

[24] This version of a standard no-bubble result confirms as a special case our ealier assertion that if A_{N+1} gets x he will consume it in the quasi-stationary model with $\gamma_n = 0$ for $n > N + 1$.

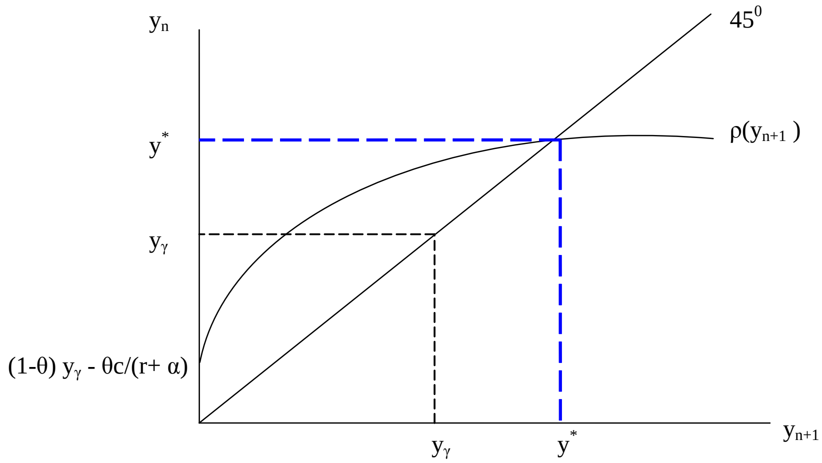


Figure 8: Best-response condition (with nonlinear utility)

in the linear case $U(y) = y$.[25] Since $U(y^*) > \gamma$ is necessary to satisfy the search viability condition $\gamma \leq [rU(y^*) - c] / (r + \alpha)$, we at least have a chance, which we did not have with $U(y) = y$.

We now show, by way of an explicit example, that search can be viable as long as γ and c are not too big. Suppose $U(y) = \sqrt{y}$, which means $y_\gamma = \gamma^2$. Setting $c = 0$, for now, we have

$$\rho(y) = (1-\theta)\gamma^2 + \frac{\theta\alpha}{r+\alpha}\sqrt{y}.$$

To find the steady state, rewrite $y = \rho(y)$ in terms of $U = \sqrt{y}$:

$$U^2 - \frac{\theta\alpha}{r+\alpha}U - (1-\theta)\gamma^2 = 0.$$

This is a quadratic in U, and the correct (i.e., positive) root is

$$U = \frac{1}{2}\left\{\frac{\theta\alpha}{r+\alpha} + \sqrt{\left(\frac{\theta\alpha}{r+\alpha}\right)^2 + 4(1-\theta)\gamma^2}\right\}.$$

The search viability condition $\gamma \leq U\alpha/(r+\alpha)$ can now be reduced to

$$\gamma \leq \frac{\theta\alpha^2}{r^2 + 2r\alpha + \theta\alpha^2} = \bar{\gamma}. \tag{25}$$

[25] As drawn, Figure 8 shows the existence of a unique positive solution to $y^* = \rho(y^*)$, which is true iff $0 < \rho(0) = (1-\theta)y_\gamma - \theta c/(r+\alpha)$, which holds as long as c or θ is small or α is big.

Since $\bar{\gamma} > 0$, search is viable for some $\gamma > 0$.

Summarizing, for some $\gamma < \bar{\gamma}$, and $c = 0$, we have constructed equilibrium where every A_n searches, and trades x to A_{n+1} for $y_n = y^*$, with no one ever consuming x. By continuity, this is also true for $c > 0$ but not too big.[26] In such an equilibrum, x circulates forever. For this to be viable we require $[\alpha U(y^*) - c] / (r + \alpha) \geq \gamma$, and *a fortiori* $y^* > y_\gamma$, so the amount of y required to acquire x is above the fundamental value. And since the surplus from trade between A_n and A_{n+1} is $S^* = U(y^*) - y^* > 0$, by definition this a genuine bubble. Once this is understood it should be clear – again by continuity – that we can generate similar bubbles when $\gamma_n > \gamma_{n+1}$ for some n, or $\gamma_n > 0 > \gamma_{n+1}$ for some n, or $\gamma_n = 0$ for all n, or even $\gamma_n < 0$ for all n, all of which may appear anomalous in other contexts. Of course, these results rely on what Shell (1971) dubbed "the economics of infinity," but in no other way are they fantastic or exotic.[27]

Having seen these stationary equilibria, one might ask if there can also be nonstationary bubbles? When $\rho(0) > 0$, Figure 8 implies the answer is no. With $\rho(0) > 0$, all paths satisfying (24), other than $y_n = y^*$ for all n, either lead to $y_n < 0$ or $y_n > \bar{y}$. But suppose $\rho(0) < 0$, as in Figure 9, which occurs whenever $c > y_\gamma (r + \alpha)(1 - \theta)/\theta$. As long as c is not too big, there are multiple steady states, y_1^* and y_2^*. Supposing $\alpha [U(y_1^*) - c] / (r + \alpha) > \gamma$, so that search is viable when y_n is near y_1^*, then as shown there are nonconstant paths for y_n

[26]Without the assumption $c = 0$, search is viable iff $Q(\gamma) \geq 0$, where $Q(\cdot)$ is the quadratic

$$Q(\gamma) = -\gamma^2[r^2 + 2r\alpha + \alpha^2\theta] + \gamma[\alpha^2\theta - 2(r + \alpha)c] - c^2.$$

Hence, $\exists \bar{c} > 0$ such that $c < \bar{c}$ implies search is viable for $\gamma \in [\gamma_1, \gamma_2]$, with $0 < \gamma_1 < \gamma_2$; and for $c > \bar{c}$ search is not viable for any $\gamma \geq 0$. As $c \to 0$, $[\gamma_1, \gamma_2] \to [0, \bar{\gamma}]$ consistent with (25).

[27]Indeed, the results should be unsurprising to those versed in monetary theory, where objects can be valued for their use in the exchange process over and above their use as commodities. Obviously, examples abound, but a classic case concerns cigarettes. In Radford's (1945) description of a POW camp, e.g., "Most trading was for food against cigarettes or other food stuffs, but cigarettes rose from the status of a normal commodity to that of currency. ... With this development everyone, including nonsmokers, was willing to sell for cigarettes, using them to buy at another time and place." Similarly, Friedman (1992) reports that "After World War II [in Germany] the Allied occupational authorities exercised sufficiently rigid control over monetary matters, in the course of trying to enforce price and wage controls, that it was difficult to use foreign currency. Nonetheless, the pressure for a substitute currency was so great that cigarettes and cognac emerged as substitute currencies and *attained an economic value far in excess of their value purely as goods to be consumed*" (empahsis added). This is exactly what is going on here when $y^* > y_\gamma$. As he goes on to say, "Foreigners often expressed surprise that Germans were so addicted to American cigarettes that they would pay a fantastic price for them. The usual reply was 'Those aren't for smoking; they're for trading'."

satisfying all the equilibrium conditions, even though fundamentals are stationary. Starting to the left of y^*, y_n rises over time in progressively smaller increments, until settling at y_1^*; starting from the right, y_n falls in progressively smaller increments, again settling at y_1^*. Hence, whether one says prices are rising or falling depends on where one starts – and, we hasten to add, on whether one takes x or y to be money, as discussed in the previous section. In any case, the outcomes are all quite bubbly.

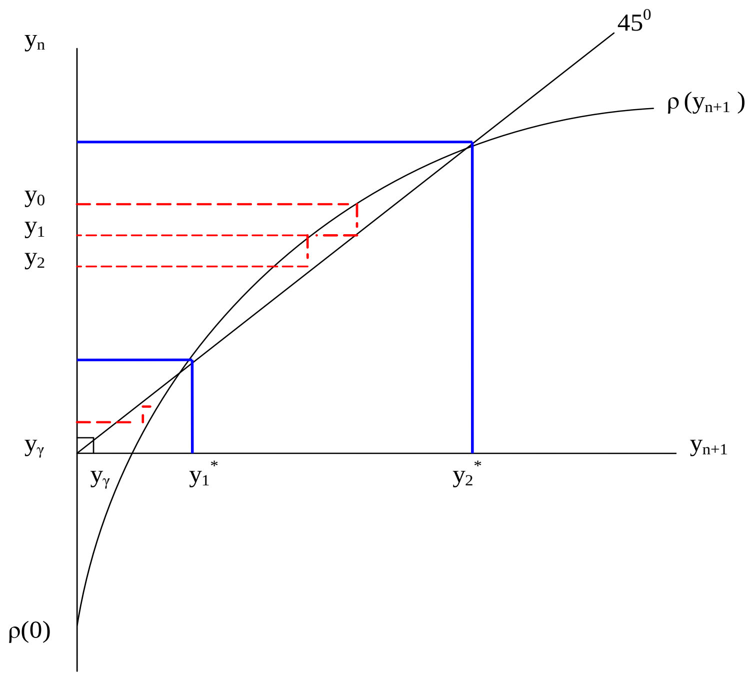


Figure 9: Best-response condition (with nonlinear utilityand $c > 0$)

We conclude the following: vis a vis off-the-shelf models of intermedation, along the lines of Rubinstein-Wolinsky, all it takes to get interesting dynamic equilibria with stationary fundamentals is nonlinear utility (in addition to the infinite horizon, which most of the models have anyway). The nonlinearity is crucial for understanding certain aspects of behavior here, such as the fact that, even with perfect foresight, our agents sometimes engage in "buying high and selling low." Consider starting to the right f y_1^* in Figure 9, where in equilibrium A_{n+1} gives up y_n to get x, and later gets $y_{n+1} < y_n$ in return. This is ostensibly a funny strategy for a middleman, even without accounting for his time and search costs, but it is actually a good deal, since $U(y_{n+1})$ exceeds y_n by enough of a margin, even if $y_{n+1} < y_n$. For this to work we need nonlinear utility. But now that we have let the nonlinear cat out of the bag, we need to revisit our bargaining solution, and ask how it compares to others.

First, it is not necessary to use our particular extensive form to generate interesting dynamics. Consider instead generalized Nash:

$$y_n = \arg\max_y [U(y) - \gamma]^\theta (V_{n+1} - y)^{1-\theta}$$

The FOC is $\theta (V_{n+1} - y_n) U'(y) = (1-\theta)[U(y_n) - \gamma]$, which has a unique solution that does not depend on n for any given V_{n+1}, say $y_n = \hat{y}(V_{n+1})$. Inserting V_{n+1} we get a dynamical system analogues to (24):

$$y_n = \hat{y}\left[\frac{\alpha U(y_{n+1}) - c}{r+\alpha}\right] = \hat{\rho}(y_{n+1}).$$

This has qualitatively similar properties to the system $y_n = \rho(y_{n+1})$ derived from our game. The same is true for Kalai's proportional solution, which satisfies

$$\theta (V_{n+1} - y) = (1-\theta)[U(y) - \gamma],$$

defining $y_n = \tilde{y}(V_{n+1})$ and $y_n = \tilde{\rho}(y_{n+1})$.

The three bargaining solutions imply different values of y_n but all generate dynamical systems that behave similarly, and all can generate multiple stationary equilibria as well as paths where y_n varies with n. We like our game because it has explicit strategic foundations.[28] This is often said to also be a desirable property of Nash bargaining: write down a standard strategic model with randomly alternating counteroffers, take the limit as the time between counteroffers goes to 0, and out jumps the Nash solution, at least in stationary bargaining situations (Binmore 1987; Binmore, Rubinstein and Wolinsky 1986). As demonstrated in Coles and Wright (1998) and Coles and Muthoo (2003), however, this breaks down in nonstationary situations unless one makes additional assumptions, like imposing linear utility or giving one agent all the bargaining power. In general, in a nonstationary

[28] Nash himself argued that it was important to write down explicit economic environments where the bargaining solution would arise as an equilibrium, an endeavor now referred to as the Nash program (e.g., see Osborne and Rubinstein 1990, Chapter 4).

bargaining situation, when one writes down the same strategic model, and takes the limit, one gets a differential equation for the terms of trade that equals Nash in steady state but not out of steady state, unless (in our notation) $U(y) = y$ or $\theta = 1$.

The above-mentioned papers also show that the set of dynamic equilibria can be qualitatively different when one uses the correct limit of the game rather than sticking in the Nash solution out of steady state. Furthermore, they show that sticking Nash into the model out of steady state is tantamount to using the equilibrium outcome of an alternating-offer game where the players have *myopic expectations* – they believe continuation values will not change over time even though in equilibrium they do change over time. To the extent that one wants strategic foundations for bargaining, and wants to analyze dynamic models out of steady state, and wants to use nonlinear utility, this is an issue. Now, we saw earlier that when utility is linear, our game is equivalent to proportional or Nash bargaining. We now compare the bargaining solutions in steady state for the nonlinear example $U(y) = \sqrt{y}$, where we emphasize that, because we are looking only at steady state, the Nash solution does follow as the limit of a strategic bargaining game, just not our strategic bargaining game.

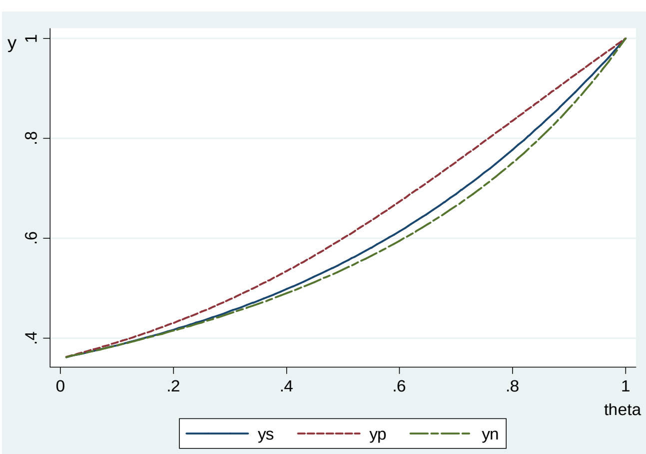
In Appendix B we solve this example with our bargaining solution, the Nash solution, and the proportional solution, and show that for each case there is an upper bounds for γ that makes search viable. When $c = 0$, e.g., these bounds are

$$\begin{aligned} \overline{\gamma}_s &= \frac{\theta\alpha^2}{r^2 + 2r\alpha + \theta\alpha^2} \\ \overline{\gamma}_n &= \frac{\theta\alpha^2}{r^2(2-\theta) + 2r\alpha + \theta\alpha^2} \\ \overline{\gamma}_p &= \frac{\alpha[\theta\alpha - r(1-\theta)]}{\theta\,(r+\alpha)^2}, \end{aligned} \tag{26}$$

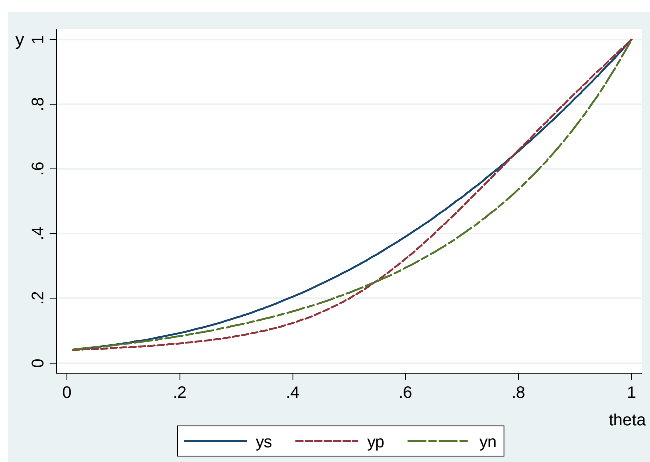
where the subscripts s, n, and p signify strategic, Nash and proportional bargaining, and we assume here that $\theta\alpha > r(1-\theta)$. One can show $\overline{\gamma}_n \leq \overline{\gamma}_s < 1$ and $\overline{\gamma}_p < 1$, but the relationship between $\overline{\gamma}_p$ and the other two is unclear, in general. However, if $\theta = 1$, they all reduce to $\overline{\gamma} = \alpha^2/(r+\alpha)^2$. Also, if $r \to 0$, they all equal 1, independent of θ, and steady state equilibrium payoffs are

$$
\begin{aligned}
U_s^* &= \frac{1}{2}[\theta + \sqrt{\theta^2 + 4(1-\theta)\gamma^2}] \\
U_n^* &= \frac{1}{2-\theta}[2\gamma(1-\theta) + \theta] \\
U_p^* &= \frac{1}{2\theta}[2\theta - 1 + \sqrt{1 - 4\theta(1-\gamma)(1-\theta)}].
\end{aligned} \tag{27}
$$

We are interested in comparing U^*, or equivalently y^*, across bargaining solutions. One can show $y_n^* < y_s^* < 1$ for $\theta < 1$ and $y_n^* = y_s^* = 1$ for $\theta = 1$. The relationship with y_p^* is less clear. The following graphs show y^* as a function of θ for two examples with different values of γ, illustrating how the solutions are noncomparable, in general. We conclude that, with nonlinear utility, even when we look only at steady states, the different bargaining solutions generally give different answers. Again, we like our solution because it has a simple strategic foundation, in and out of steady state. We want to look at models out of steady state because we are interested in nonstationary bubbles, and we need to look at nonlinear utility because that is the only way to get such outcomes. That is why we used this bargaining solution, that and the fact that it is so simple, but one can use other bargaining solutions and get similar results. The model of intermediated trade, or, from a different perspective, the model of an indivisible asset being used as a medium of exchange, works fine in any case.



Model comparison when $\gamma = 0.6$



Model comparison when $\gamma = 0.2$

7 Conclusion

We conclude as we began, by saying that we are interested in trade that may be intermediated by middlemen. We showed how to get middlemen, and sometimes long chains of middlemen, even infinite chains, with no end user in sight, that look like bubbles. This arose naturally from our discussion of buyers versus sellers and goods versus money that itself arose from our reading of the middleman and money literatures. There has been much progress on these topics, much of it based on search theory and related techniques (the economics of information), which is not surprising given money and middlemen are both institutions that unquestionably exist to ameliorate frictions in the process of exchange. What is surprising is that there is not much connection between the literatures, although there are a few papers that discuss some of the issues, including Li (1999). We think we have something to contribute by highlighting the similarities and differences between existing models, and by raising some new issues.

One new issue on which we focus is the phenomenon of intermediation chains, where A sells to B who sells to C and so on. A related issue concerns fleshing out ideas about bargaining with bargainers, leading to further insights on the ubiquity of holdup problems. We also presented a comparison of various bargaining solutions. We think the strategic bargaining solution on which we mainly focus is an attractive one for search models, especially those with nonlinear utility and nonstationary equilibria, as in monetary theory. Many applications of these ideas are potentially interesting, including applications in financial markets, and perhaps especially in real estate markets, where flipping is a prevalent activity. Additional research might also look into why producers often trade only with large or established wholesalers, stymieing efforts to disintermediate, or, to cut out the middleman. It would also be interesting to try to understand further how buyers and sellers are different and why they are often treated so differently, as they are by the legal system. We leave all of this to future research.

Appendix A

Consider a mixed-strategy equilibrium where $\sigma_1 \in (0,1)$ and $V_1 = 0$. For convenience, define $n = 1 - m$. Then write the steady state condition (12) as $\sigma_1 = \sigma(n)$ and write $V_1 = 0$ as $\sigma_1 = \Sigma(n)$, where

$$\sigma_1 = \sigma(n) = \frac{\pi_2 \alpha_{23}(1-n)}{\pi_1 \alpha_{12} n} \tag{28}$$

$$\sigma_1 = \Sigma(n) = \frac{\alpha_{12}\theta_{12} n(\bar{c}_2 - c_2) - (r + \alpha_{23})(c_1 - \bar{c}_1)}{\alpha_{21}(1-\theta_{12})(c_1 - \bar{c}_1)}. \tag{29}$$

When $\sigma_1 = 1$ (28) implies $n = n_1 = \pi_2\alpha_{23}/(\pi_2\alpha_{23} + \pi_1\alpha_{12})$. As long as $\Sigma(1) > 0$ and $\Sigma(n_1) < 1$, there exists a unique $(n^*, \sigma^*) \in (n_1, 1) \times (0,1)$ such that $\sigma(n^*) = \Sigma(n^*) = \sigma^*$. We claim that $\Sigma(1) > 0$ iff $g(c_1) \geq c_2$ and $\Sigma(n_1) < 1$ iff $h(c_1) \leq c_2$ at $n = n_1$. See Figure 10. The best response conditions for $\mu = 1$ and $\sigma_2 = 1$ are not binding in this case. This completes the argument. ■

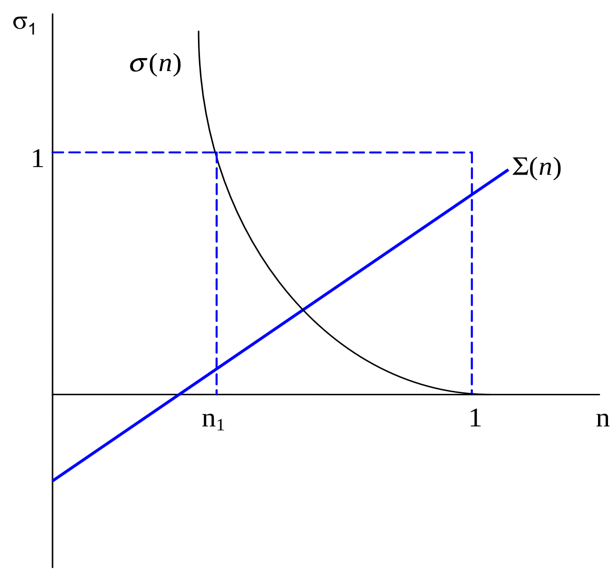


Figure 10: Existence of (n^*, σ^*)

Appendix B

Here we give some more details concerning the example with $U(y) = \sqrt{y}$. First, we can extend the case of our bargaining solution by relaxing the assumption $c = 0$ made in the text. One can show search is viable iff $Q(\gamma) \geq 0$, where $Q(\cdot)$ is the quadratic

$$Q(\gamma) = -\gamma^2[r^2 + 2r\alpha + \alpha^2\theta] + \gamma[\alpha^2\theta - 2(r+\alpha)c] - c^2.$$

Hence, $\exists \bar{c} > 0$ such that $c < \bar{c}$ implies search is viable for $\gamma \in [\gamma_1, \gamma_2]$, with $0 < \gamma_1 < \gamma_2$; and for $c > \bar{c}$ search is not viable for any $\gamma \geq 0$. As $c \to 0$, $[\gamma_1, \gamma_2] \to [0, \bar{\gamma}]$ consistent with (25).

Now consider the FOC from generalized Nash bargaining, $\theta y = \theta V - (1-\theta)[\sqrt{y} - \gamma]2\sqrt{y}$. Substituting V and rearranging terms, the steady state y solves

$$(2-\theta)(r+\alpha)y - [2\gamma(1-\theta)(r+\alpha) + \alpha\theta]\sqrt{y} + c\theta = 0.$$

The solution satisfies

$$\sqrt{y} = \frac{[2\gamma(1-\theta)(r+\alpha) + \alpha\theta] + \sqrt{[2\gamma(1-\theta)(r+\alpha) + \alpha\theta]^2 - 4(2-\theta)(r+\alpha)c\theta}}{2(2-\theta)(r+\alpha)}$$

Inserting $U = \sqrt{y}$ into the viability condition $\gamma \leq (\alpha U - c)/(r+\alpha)$ and simplifying, we have

$$\gamma^2[r^2(2-\theta) + 2r\alpha + \alpha^2\theta] + \gamma\{[r(2-\theta) + \alpha]2c - \alpha^2\theta\} + (2-\theta)c^2 \leq 0.$$

Again there exists $\bar{c} > 0$ such that $c < \bar{c}$ implies search is viable for $\gamma \in [\gamma_1, \gamma_2]$, with $0 < \gamma_1 < \gamma_2$; and for $c > \bar{c}$ search is not viable for $\gamma \geq 0$.

One can do the same for proportional bargaining. At steady state, we have

$$\theta y + \frac{[(1-\theta)r + \alpha]}{r+\alpha}\sqrt{y} + [\frac{c\theta}{r+\alpha} - \gamma(1-\theta)] = 0.$$

The solution satisfies

$$\sqrt{y} = \frac{1}{2\theta}\left\{-[(1-\theta)r + \alpha]/(r+\alpha) + \sqrt{\frac{[(1-\theta)r+\alpha]^2}{(r+\alpha)^2} - 4\theta[c\theta/(r+\alpha) - \gamma(1-\theta)]}\right\},$$

and the viability condition is

$$\gamma^2 (r+\alpha)^2 \theta + \gamma\{2\theta c(r+\alpha) + \alpha[r(1-\theta) - \theta\alpha]\} + c[\theta c + \alpha(1-\theta)] \leq 0.$$

Again, search is viable iff c is small.

When $c = 0$, the upper bounds for γ that allow search are as given in (26). Algebra implies $\overline{\gamma}_n \leq \overline{\gamma}_s < 1$ and $\overline{\gamma}_p < 1$, while the relationship between $\overline{\gamma}_p$ and the other two is ambiguous. When $c = 0$ and $r \to 0$, all $\overline{\gamma}_n = \overline{\gamma}_s = \overline{\gamma}_p = 1$ independent of θ, and in equilibrium U is given by (27). ■

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